ESERCIZIO 1

PROBLEMA

Si ricordi che una regola di *deduzione* è un termine che ha la struttura:

regola(<sigla>,<lista antecedenti>,<conseguente>).

Tale termine indica una regola di nome <sigla> che consente di dedurre <conseguente> conoscendo tutti gli elementi contenuti nella <lista antecedenti>, detta anche *premessa*.

Un *procedimento di deduzione* (o deduttivo, o di calcolo) è rappresentato da un *insieme di regole da applicare in sequenza opportuna* per dedurre un certo elemento (incognito) a partire da certi dati: quindi può essere descritto dalla lista delle sigle di queste regole.

In tale lista non ci sono regole *ripetute*; l'applicazione di una regola rende disponibile il conseguente da utilizzare (come antecedente) nell'applicazione di regole successive.

Nelle liste richieste le sigle delle regole sono elencate nell'ordine che corrisponde alla sequenza di applicazione: la prima (a sinistra) della lista deve essere la sigla che corrisponde alla prima regola da applicare (che ha come antecedenti solo dati); l'ultima (a destra) deve essere la sigla della regola che ha come conseguente l'elemento incognito da dedurre.

Per rendere unica la lista associata a un ben preciso procedimento (cioè a un ben preciso insieme di regole), si costruisce tale lista per passi successivi a partire dal primo elemento che è la sigla della prima regola da applicare; ad ogni passo, se ci fossero più regole applicabili, *per quel procedimento*, occorre dare la precedenza (nella lista) alla regola con sigla *inferiore*.

In alcuni casi esistono *più procedimenti* deduttivi possibili che permettono di ricavare un certo elemento dagli stessi dati in maniere diverse (cioè con *insiemi diversi di regole*).

Sono date le seguenti regole:

regola(1,[q,z],n)	regola(2,[a,p],x)	regola(3,[t],q)	regola(4,[a,n,x],b)
regola(5,[a,n,q,z],b)	regola(6,[r,u],b)	regola(7,[n,q,z],a)	regola(8,[w],q)
regola(9,[x,a],n)	regola(10,[r,t],a)	regola(11,[a,r,q],u)	regola(12,[y],p)

Trovare:

1. la lista L1 che descrive il procedimento per dedurre **b** conoscendo **w** e **z**;
2. la lista L2 che descrive il procedimento per dedurre **b** conoscendo **a** e **y**;
3. la lista L3 che descrive il procedimento per dedurre **b** conoscendo **r** e **t**;
4. la lista L4 che descrive il procedimento per dedurre **b** conoscendo **a**, **r** e **w**

L1	[ ]
L2	[ ]
L3	[ ]
L4	[ ]

SOLUZIONE

L1	[8,1,7,5]
L2	[12,2,9,4]
L3	[3,10,11,6]
L4	[8,11,6]

COMMENTI ALLA SOLUZIONE

La incognita comune alle 4 domande, **b**, è deducibile con tre regole: 4, 5 e 6, che hanno come antecedenti, rispettivamente, [a,n,x], [a,n,q,z], [r,u].

Per la prima domanda, visto che è noto **z** e **q** è immediatamente deducibile da **w** (con la regola 8), conviene provare ad usare la regola 5 per dedurre **b**; occorre dedurre **n** (e questo è possibile con la regola 1, visto che **z** e **q** sono noti) e **a** (e questo è possibile con la regola 7 (da n, q, z). Il procedimento è quindi [8,1,7,5].
Per la seconda domanda, visto che è noto **a** le regole più probabili per dedurre **b** sono la 4 e la 5. È facile decidere in favore della prima osservando che da **y** si deduce **p** (con la regola 12) poi **x** (con la regola 2) e infine **n** (con la regola 9). Il procedimento è quindi [12,2,9,4].
Per la terza domanda, essendo noto **r**, la regola 6 è la naturale candidata per dedurre **b**. Infatti, applicando le regole 10, 3 e 11 si deducono rispettivamente **a**, **q** e **u**. Il procedimento (rispettando la precedenza tra regole) è quindi [3,10,11,6].
Per la quarta domanda ci si può ricondurre alla terza, osservando che inizialmente la 8 è l'unica regola applicabile; poi, ottenuto **q** si procede a dedurre a con la regola 11. Il procedimento completo è [8,11,6].

ESERCIZIO 2

PROBLEMA

In un deposito di minerali esistono esemplari di vario peso e valore individuati da sigle di riconoscimento. Ciascun minerale è descritto da un termine che contiene le seguenti informazioni:

minerale(<sigla del minerale>, <valore in euro>, <peso in Kg>).

Il deposito contiene i seguenti minerali:

minerale(m1,591,899)	minerale(m2,536,864)	minerale(m3,587,833)
minerale(m4,562,858)	minerale(m5,545,825)	minerale(m6,558,842)
minerale(m7,573,877)	minerale(m8,594,886)	minerale(m9,529,891)

Disponendo di un autocarro con portata massima di 1700 Kg, trovare la lista L1 delle sigle di 2 minerali diversi trasportabili con questo autocarro che consente di trasportare il massimo valore possibile.
Disponendo di un autocarro con portata massima di 2550 Kg, trovare la lista L2 delle sigle di 3 minerali diversi trasportabili con questo autocarro che consente di trasportare il massimo valore possibile.
Disponendo di un autocarro con portata massima di 3400 Kg, trovare la lista L3 delle sigle di 4 minerali diversi trasportabili con questo autocarro che consente di trasportare il massimo valore possibile.

N.B. Nelle liste, elencare le sigle in ordine crescente; per le sigle si ha il seguente ordine:
m1<m2<… <m9.

L1	[ ]
L2	[ ]
L3	[ ]

SOLUZIONE

L1	[m3,m4]
L2	[m3,m5,m8]
L3	[m3,m5,m6,m8]

COMMENTI ALLA SOLUZIONE

In generale, un metodo per risolvere il problema (detto della *forza bruta*) è quello di generare tutte le combinazioni di 2, 3 e 4 minerali presi tra i nove del deposito, calcolarne peso e valore e scegliere, tra quelle trasportabili, quella che ha valore maggiore; poiché tali combinazioni sono rispettivamente (9×8)/(2×1) = 36, (9×8×7)/(3×2×1) = 84, e (9×8×7×6)/(4×3×2×1) = 126 tale metodo è pesante (cioè richiede molti “calcoli” e molto “spazio”).
Le combinazioni di 2 minerali, con il valore e il peso, sono:

[m1,m2]	[1127,1763]	[m1,m9]	[1120,1790]	[m2,m9]	[1065,1755]
[m1,m3]	[1178,1732]	[m2,m3]	[1123,1697] t	[m3,m4]	[1149,1691] t
[m1,m4]	[1153,1757]	[m2,m4]	[1098,1722]	[m3,m5]	[1132,1658] t
[m1,m5]	[1136,1724]	[m2,m5]	[1081,1689] t	[m3,m6]	[1145,1675] t
[m1,m6]	[1149,1741]	[m2,m6]	[1094,1706]	[m3,m7]	[1160,1710]
[m1,m7]	[1164,1776]	[m2,m7]	[1109,1741]	[m3,m8]	[1181,1719]
[m1,m8]	[1185,1785]	[m2,m8]	[1130,1750]	[m3,m9]	[1116,1724]

[m4,m5]	[1107,1683] t	[m5,m6]	[1103,1667] t	[m6,m8]	[1152,1728]
[m4,m6]	[1120,1700] t	[m5,m7]	[1118,1702]	[m6,m9]	[1087,1733]
[m4,m7]	[1135,1735]	[m5,m8]	[1139,1711]	[m7,m8]	[1167,1763]
[m4,m8]	[1156,1744]	[m5,m9]	[1074,1716]	[m7,m9]	[1102,1768]
[m4,m9]	[1091,1749]	[m6,m7]	[1131,1719]	[m8,m9]	[1123,1777]

Le combinazioni di 3 minerali, con il valore e il peso, sono:

[m1,m2,m3][1714,2596]	[m2,m3,m4][1685,2555]	[m3,m5,m8][1726,2544] t
[m1,m2,m4][1689,2621]	[m2,m3,m5][1668,2522] t	[m3,m5,m9][1661,2549] t
[m1,m2,m5][1672,2588]	[m2,m3,m6][1681,2539] t	[m3,m6,m7][1718,2552]
[m1,m2,m6][1685,2605]	[m2,m3,m7][1696,2574]	[m3,m6,m8][1739,2561]
[m1,m2,m7][1700,2640]	[m2,m3,m8][1717,2583]	[m3,m6,m9][1674,2566]
[m1,m2,m8][1721,2649]	[m2,m3,m9][1652,2588]	[m3,m7,m8][1754,2596]
[m1,m2,m9][1656,2654]	[m2,m4,m5][1643,2547] t	[m3,m7,m9][1689,2601]
[m1,m3,m4][1740,2590]	[m2,m4,m6][1656,2564]	[m3,m8,m9][1710,2610]
[m1,m3,m5][1723,2557]	[m2,m4,m7][1671,2599]	[m4,m5,m6][1665,2525] t
[m1,m3,m6][1736,2574]	[m2,m4,m8][1692,2608]	[m4,m5,m7][1680,2560]
[m1,m3,m7][1751,2609]	[m2,m4,m9][1627,2613]	[m4,m5,m8][1701,2569]
[m1,m3,m8][1772,2618]	[m2,m5,m6][1639,2531] t	[m4,m5,m9][1636,2574]
[m1,m3,m9][1707,2623]	[m2,m5,m7][1654,2566]	[m4,m6,m7][1693,2577]
[m1,m4,m5][1698,2582]	[m2,m5,m8][1675,2575]	[m4,m6,m8][1714,2586]
[m1,m4,m6][1711,2599]	[m2,m5,m9][1610,2580]	[m4,m6,m9][1649,2591]
[m1,m4,m7][1726,2634]	[m2,m6,m7][1667,2583]	[m4,m7,m8][1729,2621]
[m1,m4,m8][1747,2643]	[m2,m6,m8][1688,2592]	[m4,m7,m9][1664,2626]
[m1,m4,m9][1682,2648]	[m2,m6,m9][1623,2597]	[m4,m8,m9][1685,2635]
[m1,m5,m6][1694,2566]	[m2,m7,m8][1703,2627]	[m5,m6,m7][1676,2544] t
[m1,m5,m7][1709,2601]	[m2,m7,m9][1638,2632]	[m5,m6,m8][1697,2553]
[m1,m5,m8][1730,2610]	[m2,m8,m9][1659,2641]	[m5,m6,m9][1632,2558]
[m1,m5,m9][1665,2615]	[m3,m4,m5][1694,2516] t	[m5,m7,m8][1712,2588]
[m1,m6,m7][1722,2618]	[m3,m4,m6][1707,2533] t	[m5,m7,m9][1647,2593]
[m1,m6,m8][1743,2627]	[m3,m4,m7][1722,2568]	[m5,m8,m9][1668,2602]
[m1,m6,m9][1678,2632]	[m3,m4,m8][1743,2577]	[m6,m7,m8][1725,2605]
[m1,m7,m8][1758,2662]	[m3,m4,m9][1678,2582]	[m6,m7,m9][1660,2610]
[m1,m7,m9][1693,2667]	[m3,m5,m6][1690,2500] t	[m6,m8,m9][1681,2619]
[m1,m8,m9][1714,2676]	[m3,m5,m7][1705,2535] t	[m7,m8,m9][1696,2654]

Le combinazioni di 4 minerali, con il valore e il peso, sono:

[m1,m2,m3,m4][2276,3454]	[m1,m2,m5,m8][2266,3474]
[m1,m2,m3,m5][2259,3421]	[m1,m2,m5,m9][2201,3479]
[m1,m2,m3,m6][2272,3438]	[m1,m2,m6,m7][2258,3482]
[m1,m2,m3,m7][2287,3473]	[m1,m2,m6,m8][2279,3491]
[m1,m2,m3,m8][2308,3482]	[m1,m2,m6,m9][2214,3496]
[m1,m2,m3,m9][2243,3487]	[m1,m2,m7,m8][2294,3526]
[m1,m2,m4,m5][2234,3446]	[m1,m2,m7,m9][2229,3531]
[m1,m2,m4,m6][2247,3463]	[m1,m2,m8,m9][2250,3540]
[m1,m2,m4,m7][2262,3498]	[m1,m3,m4,m5][2285,3415]
[m1,m2,m4,m8][2283,3507]	[m1,m3,m4,m6][2298,3432]
[m1,m2,m4,m9][2218,3512]	[m1,m3,m4,m7][2313,3467]
[m1,m2,m5,m6][2230,3430]	[m1,m3,m4,m8][2334,3476]
[m1,m2,m5,m7][2245,3465]	[m1,m3,m4,m9][2269,3481]

[m1,m3,m5,m6][2281,3399] t
[m1,m3,m5,m7][2296,3434]
[m1,m3,m5,m8][2317,3443]
[m1,m3,m5,m9][2252,3448]
[m1,m3,m6,m7][2309,3451]
[m1,m3,m6,m8][2330,3460]
[m1,m3,m6,m9][2265,3465]
[m1,m3,m7,m8][2345,3495]
[m1,m3,m7,m9][2280,3500]
[m1,m3,m8,m9][2301,3509]
[m1,m4,m5,m6][2256,3424]
[m1,m4,m5,m7][2271,3459]
[m1,m4,m5,m8][2292,3468]
[m1,m4,m5,m9][2227,3473]
[m1,m4,m6,m7][2284,3476]
[m1,m4,m6,m8][2305,3485]
[m1,m4,m6,m9][2240,3490]
[m1,m4,m7,m8][2320,3520]
[m1,m4,m7,m9][2255,3525]
[m1,m4,m8,m9][2276,3534]
[m1,m5,m6,m7][2267,3443]
[m1,m5,m6,m8][2288,3452]
[m1,m5,m6,m9][2223,3457]
[m1,m5,m7,m8][2303,3487]
[m1,m5,m7,m9][2238,3492]
[m1,m5,m8,m9][2259,3501]
[m1,m6,m7,m8][2316,3504]
[m1,m6,m7,m9][2251,3509]
[m1,m6,m8,m9][2272,3518]
[m1,m7,m8,m9][2287,3553]
[m2,m3,m4,m5][2230,3380] t
[m2,m3,m4,m6][2243,3397] t
[m2,m3,m4,m7][2258,3432]
[m2,m3,m4,m8][2279,3441]
[m2,m3,m4,m9][2214,3446]
[m2,m3,m5,m6][2226,3364] t
[m2,m3,m5,m7][2241,3399] t
[m2,m3,m5,m8][2262,3408]
[m2,m3,m5,m9][2197,3413]
[m2,m3,m6,m7][2254,3416]
[m2,m3,m6,m8][2275,3425]
[m2,m3,m6,m9][2210,3430]
[m2,m3,m7,m8][2290,3460]
[m2,m3,m7,m9][2225,3465]
[m2,m3,m8,m9][2246,3474]
[m2,m4,m5,m6][2201,3389] t
[m2,m4,m5,m7][2216,3424]
[m2,m4,m5,m8][2237,3433]
[m2,m4,m5,m9][2172,3438]
[m2,m4,m6,m7][2229,3441]
[m2,m4,m6,m8][2250,3450]
[m2,m4,m6,m9][2185,3455]
[m2,m4,m7,m8][2265,3485]
[m2,m4,m7,m9][2200,3490]
[m2,m4,m8,m9][2221,3499]
[m2,m5,m6,m7][2212,3408]
[m2,m5,m6,m8][2233,3417]
[m2,m5,m6,m9][2168,3422]
[m2,m5,m7,m8][2248,3452]
[m2,m5,m7,m9][2183,3457]
[m2,m5,m8,m9][2204,3466]
[m2,m6,m7,m8][2261,3469]
[m2,m6,m7,m9][2196,3474]
[m2,m6,m8,m9][2217,3483]
[m2,m7,m8,m9][2232,3518]
[m3,m4,m5,m6][2252,3358] t
[m3,m4,m5,m7][2267,3393] t
[m3,m4,m5,m8][2288,3402]
[m3,m4,m5,m9][2223,3407]
[m3,m4,m6,m7][2280,3410]
[m3,m4,m6,m8][2301,3419]
[m3,m4,m6,m9][2236,3424]
[m3,m4,m7,m8][2316,3454]
[m3,m4,m7,m9][2251,3459]
[m3,m4,m8,m9][2272,3468]
[m3,m5,m6,m7][2263,3377] t
[m3,m5,m6,m8][2284,3386] t
[m3,m5,m6,m9][2219,3391] t
[m3,m5,m7,m8][2299,3421]
[m3,m5,m7,m9][2234,3426]
[m3,m5,m8,m9][2255,3435]
[m3,m6,m7,m8][2312,3438]
[m3,m6,m7,m9][2247,3443]
[m3,m6,m8,m9][2268,3452]
[m3,m7,m8,m9][2283,3487]
[m4,m5,m6,m7][2238,3402]
[m4,m5,m6,m8][2259,3411]
[m4,m5,m6,m9][2194,3416]
[m4,m5,m7,m8][2274,3446]
[m4,m5,m7,m9][2209,3451]
[m4,m5,m8,m9][2230,3460]
[m4,m6,m7,m8][2287,3463]
[m4,m6,m7,m9][2222,3468]
[m4,m6,m8,m9][2243,3477]
[m5,m6,m7,m9][2205,3435]
[m5,m6,m8,m9][2226,3444]
[m5,m7,m8,m9][2241,3479]
[m6,m7,m8,m9][2254,3496]

ESERCIZIO 3

PROBLEMA

Paula and Joan were selling oranges and, each day, they had an equal number of fruit but Joan had larger ones and sold them at the rate of two for a dollar, while Paula sold three of hers for a dollar. Each lady expected to sell her fruits completely (with no oranges left).
Paula had to leave for a day and asked Joan to dispose of her stock. Upon accepting the responsibility of disposing of her friend's stock, Joan mixed them together and sold them off at the rate of five oranges for two dollars.
When Paula returned the next day, the oranges had all been disposed of (not one remained), but when they came to divide the money, they found that they were just *seven* dollars short with respect to the money they would have earned selling oranges separately.
Anyway, they divided the money equally, each taking one-half. Find how much money Joan lost by the unfortunate partnership.
Enter your answer, as an integer number (of dollars), in the box below. (Taking into consideration divisors and common multiples could be helpful.)

SOLUZIONE

21

COMMENTI ALLA SOLUZIONE

You can solve this problem in various ways.
Note that each lady expected to sell all her oranges; they had the same amount of oranges so, that common number should be a multiple of 2 and of 3, that means a multiple of 6.
On the other hand, when put together the oranges were sold off at the rate of five, so the total number should be a multiple of 6, of 5 and of 2 more.
So the number of oranges they have altogether is 60 or a multiple of 60.
When sold separately the 30 oranges (or a multiple) of Paula would fetch her 10 dollars (or a multiple) and the 30 oranges (or a multiple) of Joan would fetch her 15 dollars (or a multiple).
So when sold separately the oranges would fetch them 10+15=25 dollars (or a multiple) altogether. However, when sold together they would fetch them only 60×2/5=24 dollars i.e. a loss of one dollar (or a multiple).
Now, since they lost 7 dollars, we know the multiple: they had altogether 60×7=420 oranges that fetched them only 420×2/5=168 dollars that they shared evenly; that means 84 dollars for each of them. However, Joan could have sold her 420/2=210 oranges for 210/2=105 dollars, so she lost 21 dollars.
Another way to solve this problem is the following.
They lost 7 dollars altogether; suppose that each lady has x oranges, then

$$x\frac{1}{2} + x\frac{1}{3} - 2x\frac{2}{5} = 7$$
$$15x + 10x - 24x = 210$$

From this $x = 210$ easily follows.
The ladies, selling the oranges together, earned $2x\frac{2}{5} = 420\frac{2}{5} = 168$ dollars that means 84 dollars each.

Selling the oranges separately, Paula would have earned $210\frac{1}{2} = 105$ dollars, and Joan would have earned $210\frac{1}{3} = 70$ dollars. In this way, Joan earned 14 dollars more and Paula lost 21 dollars.

ESERCIZIO 4

PROBLEMA
Si consideri la seguente procedura PROVA1.

```
procedure PROVA1;
variables A, B float; K integer;
A ← 0.0;
B ← 1.0;
K ← 0;
while B > 0 do
        K ← K + 1;
        A ← A + B/16.0;
        B ← B - A/16.0;
endwhile;
output K;
endprocedure;
```

Determinare il valore di output di K.

K	

SOLUZIONE

K	25

COMMENTI ALLA SOLUZIONE
La seguente tabella mostra i valori assunti da K, A, B alla fine del corpo del ciclo “while”.

K	A	B
1	0.0625	0.99609375
2	0.124755859375	0.9882965087890625
3	0.1865243911743164	0.9766387343406677
4	0.24756431207060814	0.9611659648362547
5	0.30763718487287406	0.9419386407817001
6	0.3665083499217303	0.9190318689115919
7	0.4239478417287048	0.8925351288035479
8	0.47973128727892655	0.862551923348615
9	0.533640782488215	0.8291993744431015
10	0.5854657433909088	0.7926077654811697
11	0.635003728733482	0.7529200324353271
12	0.6820612307606899	0.710291205512784
13	0.7264544311052389	0.6648878035687066
14	0.7680099188282831	0.6168871836419388
15	0.8065653678059043	0.5664768481540698
16	0.8419701708155336	0.513853712478099

17	0.8740860278454148	0.45922333573776053
18	0.9027874863290248	0.40279911784219646
19	0.9279624311941621	0.34480146589256133
20	0.9495125228124472	0.2854569332167834
21	0.9673535811384962	0.22499733439562739
22	0.981415914538223	0.16365883973698844
23	0.9916445920217848	0.10168105273562689
24	0.9979996578177615	0.039306074122016796
25	1.0004562874503875	-0.023222443843632423

Naturalmente è molto difficile fare i conti manualmente, mentre è molto rapido eseguirli scrivendo un programma.

ESERCIZIO 5

PROBLEMA
Si consideri la seguente procedura PROVA2.

```
procedure PROVA2;
variables A, B, C, K, P integer; W float;
A ← 1;
B ← 3;
W ← 0.0;
P ← 1;
K ← 0;
while B > W do
        K ← K +1;
        C ← B;
        B ← A + B;
        A ← C;
        P ← P+P;
        W ← P/10.0;
endwhile;
output B, P, K;
endprocedure;
```

Determinare i valori di output.

B	
P	
K	

SOLUZIONE

B	5778
P	65536
K	16

COMMENTI ALLA SOLUZIONE
Alla fine del corpo del ciclo “while” le variabili K, A, B, C, P, W assumono i valori riportati nella tabella seguente.

K	A	B	C	P	W
1	3	4	3	2	0.2
2	4	7	4	4	0.4
3	7	11	7	8	0.8
4	11	18	11	16	1.6
5	18	29	18	32	3.2
6	29	47	29	64	6.4
7	47	76	47	128	12.8
8	76	123	76	256	25.6
9	123	199	123	512	51.2

10	199	322	199	1024	102.4
11	322	521	322	2048	204.8
12	521	843	521	4096	409.6
13	843	1364	843	8192	819.2
14	1364	2207	1364	16384	1638.4
15	2207	3571	2207	32768	3276.8
16	3571	5778	3571	65536	6553.6
17	5778	9349	5778	131072	13107.2

ESERCIZIO 6

PROBLEMA
Si consideri la seguente procedura PROVA3.

```
procedure PROVA3;
variables A, Q, M, J, K, N integer;
Q ← 0;
for K from 1 to 5 step 1 do
        input A;
        for J from 1 to 4 do
                M ← A;
                for N from 1 to 3 do
                        M ← M + K;
                endfor;
                Q ← Q + M + J;
        endfor;
        output Q;
endfor;
endprocedure;
```

Se i valori di input per A sono 1, 2, 3, 4, e 5 scrivere la lista L contenente nell'ordine i valori in output di Q.

L	[ ]

SOLUZIONE

L	[26,68,126,200,290]

COMMENTI ALLA SOLUZIONE
Si può procedere in due modi: simulando dettagliatamente la procedura (eventualmente con un programma) oppure ragionando sul suo "significato"; nel seguito viene illustrato il secondo modo.
Innanzitutto occorre notare che il costrutto

```
for N from 1 to 3 do
        M ← M + K;
 endfor;
```

equivale semplicemente a $M \leftarrow M + 3 \times K$; inoltre i valori di A letti di volta in volta sono quelli di K, quindi il corpo del costrutto "for" con la variabile J può essere riscritto nella maniera seguente:

```
for J from 1 to 4 do
        M ← 4 × K;
        Q ← Q + M + J;
endfor;
```

Questo, a sua volta, equivale a:

```
for J from 1 to 4 do
        Q ← Q + 4 × K + J;
endfor;
```

L'effetto netto è quello di sommare al valore di Q 4 volte la quantità $(4 \times K)$ e i numeri da 1 a 4, cioè sommare a Q la quantità $16 \times K + 10$. Quindi tutta la procedura equivale alla seguente:

```
procedure PROVA31;
```

```
variables A, Q, K integer;
Q ← 0;
for K from 1 to 5 step 1 do
        Q ← Q + 16 × K + 10;
        output Q;
endfor;
endprocedure;
```

Adesso è facile determinare l'output; i valori assunti da Q sono via via i seguenti:

$0 + 16 \times 1 + 10 = 26$

$26 + 16 \times 2 + 10 = 68$

$68 + 16 \times 3 + 10 = 126$

$126 + 16 \times 4 + 10 = 200$

$200 + 16 \times 5 + 10 = 290$

ESERCIZIO 7

PROBLEMA

One day two swimmers swim lengths in a pool that is 100 m long. They start at the same time from the south end (S) of the pool, swim to the north end (N), swim back to S, then to N, and so on. They each swim at a constant speed and each turns around instantly at both ends of the pool. The swimmers are said to *cross* when they pass each other in the pool while swimming in opposite directions. We also say that they *cross* if they *arrive* at an end (N or S) at the same time.

Suppose that two swimmers, Amanda and Bob, cross at S after Amanda has swum 200 m and Bob has swum 400 m. How many times before this point did they cross?

A day later two different swimmers, Charles and David, cross at S after Charles has swum 400 m and David has swum 600 m. How many times before this point did they cross?

Put your answers, as integer numbers, in the boxes below.

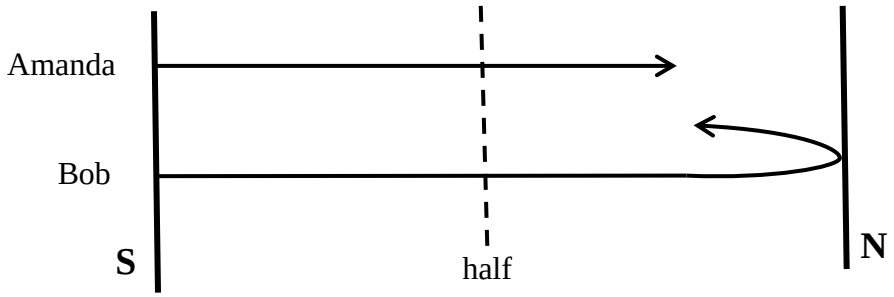
Amanda and Bob	
Charles and David	

SOLUZIONE

Amanda and Bob	2
Charles and David	4

COMMENTI ALLA SOLUZIONE

For the first question, it is easy to see that Amanda and Bob cross 2 times before finally crossing at S. Indeed, while Amanda swims the first half of the first length, Bob gets to N; then he goes back, so he has to cross her while she is completing the second half of her first length (as shown in the following figure).



The same (in a reverse pattern) happens during Amanda's second length: they cross while she is swimming the first half of her second length.

For the second question, call the two swimmers C (for Charles) and D (for David) for short. If C and D cross at S, then they have both swum an even number of lengths of the pool, so the distance that each have swum is an even multiple of 100 m. Therefore, if they cross at S, the sum of the distances that they have swum is an even multiple of 100 m.

If C and D cross at N, each one has swum an odd number of lengths of the pool, but still the sum of the distances they have swum is an even multiple of 100 m (odd plus odd is even).

A more detailed (but still easy) reasoning results in the conclusion that if the swimmers cross at any point, then the total distance swum (by both) so far is an even multiple of 100 m (which is a multiple of 200 m).

C and D have swum a total of 400 + 600 = 1000 m. The possible previous total distances at which they could have crossed are 200 m, 400 m, 600 m, and 800 m. Of course one should check that each of these distances will, in fact, yield a crossing (for example simulating what happens during each length of the slower swimmer).

ESERCIZIO 8

PROBLEMA

In a sequence of 10 terms, the first term is 1, the second term is x, and each term after the second is the sum of the previous two terms. For example, if $x = 11$, the sequence would be

1; 11; 12; 23; 35; 58; 93; 151; 244; 395.

For some values of x, the number 463 appears somewhere in the sequence. Let x be a positive integer; what is the sum of all the values of x for which 463 appears somewhere in the sequence?

Put your answer in the box below.

SOLUZIONE

1296

COMMENTI ALLA SOLUZIONE

We are given that the first two terms of a 10 term sequence are 1 and x. Since each term after the second is the sum of the previous two terms, then the third term is $1 + x$.

Since the fourth term is the sum of the second and third terms, then the fourth term is

$$x + (1 + x) = 1 + 2x.$$

Continuing in this manner, we construct the 10 term sequence:

$$1;\ x;\ 1 + x;\ 1 + 2x;\ 2 + 3x;\ 3 + 5x;\ 5 + 8x;\ 8 + 13x;\ 13 + 21x;\ 21 + 34x$$

Each term, from the second to the tenth, is dependent on the value of x, and thus, any one of these terms could potentially equal 463.

For the second term to equal 463, we need $x = 463$, which is possible since the only requirement is that x is a positive integer. Thus, if $x = 463$ then 463 appears as the second term in the sequence.

For the third term to equal 463, we need $1 + x = 463$, or $x = 462$. Thus, if $x = 462$ then 463 appears as the third term in the sequence.

For the fourth term to equal 463, we need $1 + 2x = 463$, or $2x = 462$ or $x = 231$. Thus, if $x = 231$ then 463 appears as the fourth term in the sequence.

For the fifth term to equal 463, we need $2 + 3x = 463$, or $3x = 461$ or $x = 461/3$.

However, 461/3 is not an integer, and thus, 463 cannot appear as the fifth term in the sequence.

We continue in this manner and summarize all the results in the table below.

Term	Expression	Equation	Value of x	Is x an integer?
2nd	x	$x = 463$	463	yes
3rd	$1 + x$	$1 + x = 463$	462	yes
4th	$1 + 2x$	$1 + 2x = 463$	231	yes
5th	$2 + 3x$	$2 + 3x = 463$	461/3	no
6th	$3 + 5x$	$3 + 5x = 463$	92	yes
7th	$5 + 8x$	$5 + 8x = 463$	458/8	no
8th	$8 + 13x$	$8 + 13x = 463$	35	yes
9th	$13 + 21x$	$13 + 21x = 463$	450/21	no
10th	$21 + 34x$	$21 + 34x = 463$	13	yes

Therefore, the sum of all possible integer values of x for which 463 appears in the sequence is 463 + 462 + 231 + 92 + 35 + 13 = 1296.